

# Modal Logic (80-315/80-615)

Spring 2020

Tuesday/Thursday 3:00–4:20, Baker Hall 235A

<https://canvas.cmu.edu/courses/13526>

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Office hour: Monday 6:30–7:30pm, Wean 5320

**Course Description:** This course is an introduction to mathematical modal logic and its applications in philosophy, computer science, linguistics, and economics. We begin with a rigorous development of propositional modal logic: the basic language, interpretation in relational structures, axiom systems, proofs, and validity. We prove soundness and completeness of various systems using the canonical model method, study model equivalences and expressivity results, establish the finite model property, and discuss decidability and basic complexity results. We also consider topological semantics as an alternative to relational semantics, and investigate the connection between the two. Finally, we introduce modal predicate logic, incorporating first-order quantification into the system. In the latter part of the course we turn our attention to more specialized logical systems and their applications, as determined by the interests of the class. Topics may include: epistemic and doxastic logics, multi-agent systems and the notion of common knowledge (with applications to game theory), deontic logics, logics for reasoning about counterfactuals, temporal and dynamic logics, public announcement logic, justification logic, and others.

**Recommended Texts:** *Modal Logic* by Patrick Blackburn, Maarten de Rijke, and Yde Venema; *Reasoning About Knowledge* by Ronald Fagin, Joseph Y. Halpern, Yoram Moses, and Moshe Vardi.

**Course Objectives:** The primary objective of this course is to develop *mathematical competence* in modal logic along with the ability apply this competence in useful ways. More precisely, this means being able to:

- navigate a variety of formal languages;
- interpret modalities in relational structures and explain the connection between formulas in the language and properties of the structures;
- formalize intuitions and analyze problems using modal logics;
- find axioms characterizing properties of important structures and prove completeness results.

## Grade Distribution:

Problem sets	60%
Tests	30%
Quizzes	10%

**Course Outline:** Weekly topics are subject to change; the below should only be considered a rough guideline. The evaluation schedule is fixed.

Date	Topic	Evaluation
1/14 1/16	introduction and motivation	– –
1/21 1/23	relational structures · modal languages · models and frames	quiz 1 PS1 assigned
1/28 1/30	validity · axioms and proofs · soundness	quiz 2 –
2/4 2/6	invariance and bisimulation	PS1 due; test 1 PS2 assigned
2/11 2/13	expressibility	quiz 3 –
2/18 2/20	frame definability	PS2 due; test 2 PS3 assigned
2/25 2/27	completeness via canonical models	quiz 4 –
3/3 3/5	extending completeness results	PS3 due; test 3 PS4 assigned
3/10 3/12	<i>no class (spring break)</i>	– –
3/17 3/19	bounded morphisms · unwinding	– –
3/24 3/26	filtration · decidability	PS4 due; test 4 PS5 assigned
3/31 4/2	topology	quiz 5 –
4/7 4/9	topological semantics	PS5 due; test 5 PS6 assigned
4/14 4/16	quantified modal logic <i>no class (spring carnival)</i>	quiz 6 –
4/21 4/23	topics (e.g., multi-agent epistemic logic, common knowledge)	PS6 due; test 6 PS7 assigned
4/28 4/30	topics (e.g., public announcements, doxastic logics)	quiz 7 PS7 due

**Evaluation:** Most of the evaluation is based on take-home problem sets, which are paired with in-class, 20-minute tests that take place on the day the problem set is due (which is always a Tuesday). Each test covers the same material as the corresponding problem set, but generally consists in shorter/easier problems. On those weeks where no problem set is due, there will be a 10-minute quiz based on the material covered during the previous two weeks. There is no final exam.

**Course Policies:** Problem sets are due at the beginning of class. Collaboration is allowed and encouraged; however, each student must write up their solutions independently, and clearly indicate for each question with whom they shared ideas (<http://www.cmu.edu/policies/student-and-student-life/academic-integrity.html>).

## Campus Resources

**Academic Development (AD):** Academic Development is the place to go for help with your academic work. They offer everything from Academic Counseling in study skills to Peer Tutoring. They also offer Supplemental Instruction and EXCEL Groups for select courses. Their services are designed to help both students who are having academic difficulties and those who just want to improve their performance. For more information, visit <http://www.cmu.edu/acadev>.

**Global Communications Center (GCC):** The GCC, on the ground floor of Hunt Library, provides one-on-one tutoring in written, oral, and visual communication for any student, at any level, in any discipline, at any stage of the composing process (<http://www.cmu.edu/gcc>).

**Intercultural Communications Center (ICC):** The Intercultural Communication Center helps nonnative English speakers (both international students and students who attended high school in the U.S.) develop the English language skills and cultural understanding needed to succeed at Carnegie Mellon. The center offers classes and noncredit workshops and seminars (for example, Presentation Basics, Communicating Data Effectively, and Language and Culture for Teaching (for international TAs)). For more information, visit <http://www.cmu.edu/icc>.

**Disability Services:** The Office of Disability Resources at Carnegie Mellon University has a continued mission to provide physical and programmatic campus access to all events and information within the Carnegie Mellon community. They work to ensure that qualified individuals receive reasonable accommodations as guaranteed by the Americans With Disabilities Act (ADA) and Section 504 of the Rehabilitation Act of 1973. For more information, visit: <http://www.cmu.edu/disability-resources/>.

If you have a disability and have an accommodations letter from the Disability Resources office, I encourage you to discuss your accommodations and needs with me as early in the semester as possible. I will work with you to ensure that accommodations are provided as appropriate. If you suspect that you may have a disability and would benefit from accommodations but are not yet registered with the Office of Disability Resources, I encourage you to contact them at [access@andrew.cmu.edu](mailto:access@andrew.cmu.edu).

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**Take care of yourself.** Do your best to maintain a healthy lifestyle this semester by eating well, exercising, getting enough sleep, and taking some time to relax. You can't achieve your goals if you're sick from stress or burnt out.

All of us benefit from support during times of struggle. You are not alone. There are many helpful resources available on campus; an important part of the college experience is learning how to ask for help. Asking for support sooner rather than later is usually better.

If you or anyone you know needs help, consider reaching out to a friend, faculty member, or family member you trust. Counseling and Psychological Services (CaPS) is also here to help: call 412-268-2922 and visit their website at <http://www.cmu.edu/counseling/>.